SOME POINTS CONCERNING THE SHIELDING OF THERMAL RADIATION

L. A. Vadachkoriya

UDC 535.211

General equations are proposed here which describe the effect of shields on the radiative heat transfer in a closed system consisting of two gray surfaces of any shape, and calculation formulas are derived for a few specific cases of such a system.

Shields are placed between insulated surfaces in order to reduce the radiative heat transmission between the latter. Particularly important is the shielding in high-vacuum thermally-insulated systems, where heat is transmitted primarily through radiation. High-vacuum thermal insulation has been widely used as both an industrial and a laboratory technique. A situation most often encountered is the one where two surfaces comprising a closed system are insulated from one another.

Specific examples of thermal radiation shielding are dealt with in the published literature [1, 2, 3] and questions concerning the effect of "floating" shields, i.e., shields immune to heat transmitted from extraneous sources of energy, on the radiative heat transfer are discussed [4, 5].

In this article an attempt will be made to answer, in a form convenient for practical calculations, the question concerning the effect of shields on the radiative heat transfer in a closed system of two gray surfaces. Only steady-state heat-transfer processes will be considered, where the temperatures of the bodies in question and the heat fluxes remain constant with time; all surfaces which participate in the heat transfer will be assumed to be gray, and the surface temperature of any shield will be assumed constant and equal to the temperature at which that shield would affect the total heat transfer as in the case of an actual temperature field. The thermal conductance of residual gases and thermal resistance of shield walls will be disregarded; the shape and the characteristics of extraneous sources of energy (heating due to penetrating radiation or due to electric current flow, heating or cooling by circulating liquids or gases or through mechanical contact) will not be considered as such, but an equivalent heat flux resulting from these sources will be introduced instead.

In Fig. 1 are shown two surfaces A and B ($T_A > T_B$) which make up a closed system. Between them have been placed n shields which divide the system AB into (n + 1) closed systems. The direction from A toward B is taken as positive, and the shields as well as the closed systems they form will be counted in that order. The closed system AB as it exists without shields will be denoted by the index "o."

A steady-state condition implies that each shield is at a thermal equilibrium:

$$\Phi_m = \Phi_{m-1} + Q_{m-1}, \tag{1}$$

and, by virtue of the Radiation Law, for each closed system:

$$\Phi_m = \sigma_0 \varepsilon_{\text{eq},m} H_m (T_{m-1}^4 - T_m^4). \tag{2}$$

where

$$\varepsilon_{\text{eq},m} = 1/[1 + (1/\varepsilon''_{m-1} - 1)\varphi_{m-1,m} + (1/\varepsilon'_m - 1)\varphi_{m,m-1}],$$

and

$$H_m = \varphi_{m-1,m} F''_{m-1} = \varphi_{m,m-1} F'_m.$$

Institute of Physics, Academy of Sciences of the GruzSSR, Tbilisi. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 1, No. 1, pp. 21-30, January, 1970. Original article submitted March 3, 1969.

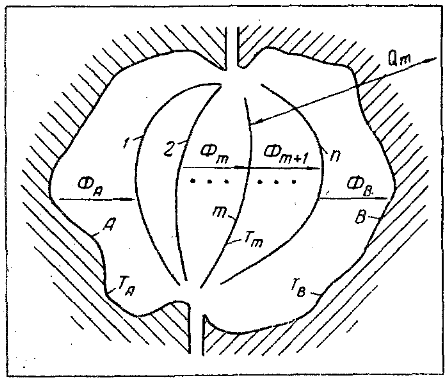


Fig. 1. Schematic diagram of a closed system consisting of two surfaces: A and B are insulated surfaces; 1 to n are shields; Φ is the resultant radiative heat flux; Q is the equivalent heat flux transmitted to a shield from extraneous sources of energy; and T (°K) is the temperature.

In our case there are n equations of the (1) kind and (n + 1) equations of the (2) kind. Their simultaneous solution will make it possible to determine the resultant radiative heat flux in each closed system and the temperature of each shield, considering also the initial conditions: the optical characteristics and the geometry of the surfaces participating in the heat transfer, the temperatures of surfaces A and B, as well as the extraneous effects on the shield.

Omitting all mathematical step-by-step transformations, we show the final equations:

$$\Phi_m = \sigma_0 \varepsilon_{AB} H_{AB} (T_A^4 - T_B^4) + \sum_{k=1}^{m-1} Q_k \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{Ak} H_{Ak}} - \sum_{k=m}^{n} Q_k \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{kB} H_{kB}} \tag{3}$$

and

$$T_m = \left[\frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{mB} H_{mB}} T_A^4 + \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{Am} H_{Am}} T_B^4 + \frac{\sum_{k=1}^{m-1} Q_k \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{Ak} H_{Ak}}}{\sigma_0 \varepsilon_{mB} H_{mB}} + \frac{\sum_{k=m}^{n} Q_k \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{kB} H_{kB}}}{\sigma_0 \varepsilon_{Am} H_{Am}} \right]^{\frac{1}{4}} \tag{4}$$

In accordance with Eq. (3), the resultant heat flux radiated from any of the (n + 1) closed systems is equal to the algebraic sum of three components. The first component of the heat flux is a consequence of the temperature difference between surfaces A and B, the second component is due to extraneous effects upon the shields located before the given system, and the third component is due to extraneous effects upon the shields located behind that system.

In accordance with Eq. (4), the temperature of any shield is determined by: the temperature of surface A, the temperature of surface B, the extraneous effects upon the shields located before that given shield, and the extraneous effects upon that given shield and the shields located behind it.

The products in (3) and (4) of the emissivities of all the closed systems and the areas of corresponding mutually-irradiating surfaces are equal to:

$$\varepsilon_{AB} H_{AB} = \frac{1}{\sum_{i=1}^{n+1} \frac{1}{\varepsilon_{\text{eq},i} H_i / H_{AB}}} H_{AB} = \frac{1}{\sum_{i=1}^{n+1} \frac{1}{\varepsilon_{\text{eq}\,i} H_i}} = \frac{1}{\frac{1}{\varepsilon_{\text{eq},1} H_1} + \cdots + \frac{1}{\varepsilon_{\text{eq},n+1} H_{n+1}}},$$
$$\varepsilon_{Am} H_{Am} = \frac{1}{\sum_{i=1}^{m} \frac{1}{\varepsilon_{\text{eq},i} H_i}}, \quad \varepsilon_{mB} H_{mB} = \frac{1}{\sum_{i=m+1}^{n+1} \frac{1}{\varepsilon_{\text{eq},i} H_i}} \quad \text{etc.} \tag{5}$$

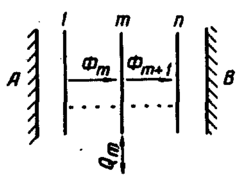
In order to determine the resultant radiative heat fluxes on surfaces A and B, the appropriate numbers for the index "m," i.e., m = 1 and m = n + 1, are now inserted into Eq. (3). After necessary transformations we will obtain:

$$\Phi_A = \Phi_1 = \sigma_0 \varepsilon_{AB} H_{AB} (T_A^4 - T_B^4) - \sum_{k=1}^{n} Q_k \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{kB} H_{kB}} \tag{6}$$

and

$$\Phi_B = \Phi_{n+1} = \sigma_0 \varepsilon_{AB} H_{AB} (T_A^4 - T_B^4) + \sum_{k=1}^{n} Q_k \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{Ak} H_{Ak}}. \tag{7}$$

TABLE 1. Calculation Formulas for the Case of Parallel Surfaces A and B Large as Compared to the Distance between Them: $\varepsilon_A = \ldots = \varepsilon'_m = \varepsilon''_m = \ldots = \varepsilon_B = \varepsilon$ and $F_A = \ldots = F'_m = F''_m = \ldots = F_B = F$



$Q_1 = \ldots = Q_n = 0$	$\Phi = \sigma_0 \frac{1}{n+1} \varepsilon_{eq} F (T_A^4 - T_B^4)$, $T'_m = \left(\frac{n-m+1}{n+1} T_A^4 + \frac{m}{n+1} T_B^4 \right)^{\frac{1}{4}}$
$Q_1 = \ldots = Q_n = Q$	$\Phi_m = \Phi + Q\left[(m-1) - \frac{n}{2}\right]$, $T_m = \left[(T'_m)^4 + \frac{Q}{\sigma_0 \varepsilon_{eq} F} m \frac{n-m+1}{2}\right]^{\frac{1}{4}}$ $\Phi_A = \Phi - Q\frac{n}{2}$, $\Phi_B = \Phi + Q\frac{n}{2}$
$Q_1 = \ldots = Q_{e-1} = Q_{e+1} = \ldots = Q_n = 0;\ Q_e \neq 0$	$\Phi_A = \Phi - Q_e \frac{n-e+1}{n+1}$; $\Phi_B = \Phi + Q_e \frac{e}{n+1}$, $e \geqslant m \begin{cases} \Phi_m = \Phi_A \\ T_m = \left[(T'_m)^4 + \frac{Q_e}{\sigma_0 \varepsilon_{eq} F} m \frac{n-e+1}{n+1}\right]^{\frac{1}{4}}, \end{cases}$ $e < m \begin{cases} \Phi_m = \Phi_B \\ T_m = \left[(T'_m)^4 + \frac{Q_e}{\sigma_0 \varepsilon_{eq} F} e \frac{n-m+1}{n+1}\right]^{\frac{1}{4}}. \end{cases}$
$Q_1 \neq \ldots \neq Q_n \neq 0$	$\Phi_m = \Phi + \frac{1}{n+1} \sum_{k=1}^{n} kQ_k - \sum_{k=m}^{n} Q_k$, $T_m = \left\{(T'_m)^4 + \frac{1}{\sigma_0 \varepsilon_{eq} F}\left[\frac{n-m+1}{n+1} \sum_{k=1}^{n-1} kQ_k + \frac{m}{n+1} \sum_{k=m}^{n} (n-k+1) Q_k\right]\right\}^{\frac{1}{4}}$, $\Phi_A = \Phi - \sum_{k=1}^{n} Q_k \frac{n-k+1}{n+1}$, $\Phi_B = \Phi + \sum_{k=1}^{n} Q_k \frac{k}{n+1}$

A steady-state condition implies that the entire system must be at a thermal equilibrium, i.e.

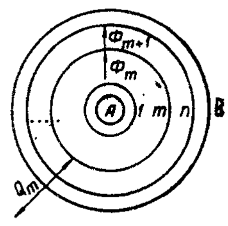
$$\Phi_B - \Phi_A = \sum_{k=1}^{n} Q_k.$$

The validity of the expression can be easily checked by substituting in it the values for Φ_A and Φ_B. The result, after minor transformations, is

$$\sum_{k=1}^{n} Q_k \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{Ak} H_{Ak}} + \sum_{k=1}^{n} Q_k \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{kB} H_{kB}} = \sum_{k=1}^{n} Q_k.$$

Here as well as in Eqs. (3), (4), (6), and (7) the dimensionless complex coefficient $\varepsilon_{AB}H_{AB}/\varepsilon_{Ak}H_{Ak}$ defines that fraction of the heat flux transmitted to a shield from extraneous sources of energy which, in

TABLE 2. Calculation Formulas for the Case of Body A and Shields 1 to n Enclosed Inside Body B with No Inner Surfaces of any Closed System thus Formed Having Concavities: $\varepsilon_A = \dots = \varepsilon'_m = \varepsilon''_m = \dots = \varepsilon_B = \varepsilon$; $F'_m = F''_m = F_m$ and $F_A/F_1 = F_1/F_2 = \dots = F_n/F_B = \varphi$

 $Q_1 = \dots = Q_n = 0$	$\Phi = \sigma_0 \frac{1-\varphi}{1-\varphi^{n+1}} \varepsilon_{eq} F_A (T_A^4 - T_B^4)$, $T'_m = \left(\frac{\varphi^m - \varphi^{n+1}}{1-\varphi^{n+1}} T_A^4 + \frac{1-\varphi^m}{1-\varphi^{n+1}} T_B^4\right)^{\frac{1}{4}}$
$Q_1 = \dots = Q_n = Q$	$\Phi_m = \Phi + Q\left[m - \frac{1}{1-\varphi} + (n+1)\frac{\varphi^{n+1}}{1-\varphi^{n+1}}\right]$ $T_m = \left[\left(T'_m\right)^4 + \frac{Q}{\sigma_0 \varepsilon_{eq} F_A} \varphi^m \frac{m-(n+1)\varphi^{n-m+1} + (n-m+1)\varphi^{n+1}}{(1-\varphi^{n+1})(1-\varphi)}\right]^{\frac{1}{4}}$. $\Phi_A = \Phi - Q\left[\frac{\varphi}{1-\varphi} - \frac{(n+1)\varphi^{n+1}}{1-\varphi^{n+1}}\right]$ $\Phi_B = \Phi + Q\left[\frac{n+1}{1-\varphi^{n+1}} - \frac{1}{1-\varphi}\right]$
$Q_1 = \dots = Q_{e-1} = Q_{e+1} = \dots = Q_n = 0;\ Q_e \neq 0$	$\Phi_A = \Phi - Q_e \frac{\varphi^e - \varphi^{n+1}}{1-\varphi^{n+1}}$, $\Phi_B = \Phi + Q_e \frac{1-\varphi^e}{1-\varphi^{n+1}}$, $e \geqslant m \begin{cases} \Phi_m = \Phi_A \\ T_m = \left[\left(T'_m\right)^4 + \frac{Q_e}{\sigma_0 \varepsilon_{eq} F_A} \frac{(\varphi^e - \varphi^{n+1})(1-\varphi^m)}{(1-\varphi^{n+1})(1-\varphi)}\right]^{\frac{1}{4}} \end{cases}$, $e < m \begin{cases} \Phi_m = \Phi_B \\ T_m = \left[\left(T'_m\right)^4 + \frac{Q_e}{\sigma_0 \varepsilon_{eq} F_A} \frac{(\varphi^m - \varphi^{n+1})(1-\varphi^e)}{(1-\varphi^{n+1})(1-\varphi)}\right]^{\frac{1}{4}} \end{cases}$
$Q_1 \neq \dots \neq Q_n \neq 0$	$\Phi_m = \Phi + \sum_{k=1}^{m-1} Q_k \frac{1-\varphi^k}{1-\varphi^{n+1}} - \sum_{k=m}^{n} Q_k \frac{\varphi^k - \varphi^{n+1}}{1-\varphi^{n+1}}$, $T_m = \left[\left(T'_m\right)^4 + \frac{1}{\sigma_0 \varepsilon_{eq} F_A}\left(\frac{\varphi^m - \varphi^{n+1}}{1-\varphi} \sum_{k=1}^{m-1} Q_k \frac{1-\varphi^k}{1-\varphi^{n+1}} + \right.\right.$ $\left.\left. + \frac{1-\varphi^m}{1-\varphi} \sum_{k=m}^{n} Q_k \frac{\varphi^k - \varphi^{n+1}}{1-\varphi^{n+1}}\right)\right]^{\frac{1}{4}}$, $\Phi_A = \Phi - \sum_{k=1}^{n} Q_k \frac{\varphi^k - \varphi^{n+1}}{1-\varphi^{n+1}}$, $\Phi_B = \Phi + \sum_{k=1}^{n} Q_k \frac{1-\varphi^k}{1-\varphi^{n+1}}$

the final analysis (as a result of radiative heat transfer), is imparted to surface B, while the ratio $\varepsilon_{AB}H_{AB}/\varepsilon_{kB}H_{kB}$ represents the fraction imparted to surface A.

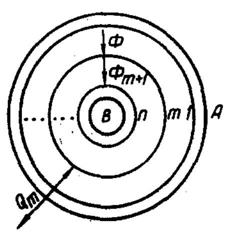
It is evident that the conditions for a most effective shielding (number of shields, their geometry and optical characteristics, also the influence of extraneous sources of energy) of surface A or B should be found from the equation $\Phi_A = 0$ or $\Phi_B = 0$ respectively.

The degree of shielding can be evaluated on the basis of the ratio $\Phi_A/\Phi_0 < 1$ for surface A and $\Phi_B/\Phi_0 < 1$ for surface B. Here

$$\Phi_0 = \sigma_0 \varepsilon_{eq,0} H_0 (T_A^4 - T_B^4)$$

is the resultant radiative heat flux transmitted from A to B when there are no shields between these two surfaces, and $H_0 = H_{AB}$.

TABLE 3. Calculation Formulas for the Case of Body B and Shields n to 1 Enclosed Inside Body A with No Inner Surface of any Closed System thus Formed Having Concavities: $\varepsilon_A = \dots = \varepsilon'_m = \varepsilon''_m = \dots = \varepsilon_B = \varepsilon$; $F'_m = F''_m = F_m$ and $F_1/F_A = F_2/F_1 = \dots = F_B/F_n = \varphi$



$Q_1 = \dots = Q_n = 0$	$\Phi = \sigma_0 \frac{1-\varphi}{1-\varphi^{n+1}} \varepsilon_{eq} F_B (T_A^4 - T_B^4)$, $T'_m = \left(\frac{1-\varphi^{n-m+1}}{1-\varphi^{n+1}} T_A^4 + \frac{\varphi^{n-m+1}-\varphi^{n+1}}{1-\varphi^{n+1}} T_B^4\right)^{\frac{1}{4}}$
$Q_1 = \dots = Q_n = Q$	$\Phi_m = \Phi + Q\left(m + \frac{\varphi}{1-\varphi} - \frac{n+1}{1-\varphi^{n+1}}\right)$, $T_m = \left[(T'_m)^4 + \frac{Q}{\sigma_0 \varepsilon_{eq} F_B} \varphi^{n-m+1} \frac{(n-m+1)-(n+1)\varphi^m + m\varphi^{n+1}}{(1-\varphi^{n+1})(1-\varphi)}\right]^{\frac{1}{4}}$, $\Phi_A = \Phi - Q\left(\frac{n+1}{1-\varphi^{n+1}} - \frac{1}{1-\varphi}\right)$, $\Phi_B = \Phi + Q\left[\frac{\varphi}{1-\varphi} - \frac{(n+1)\varphi^{n+1}}{1-\varphi^{n+1}}\right]$
$Q_1 = \dots = Q_{e-1} = Q_{e+1} = \dots = Q_n = 0;\ Q_e \neq 0$	$\Phi_A = \Phi - Q_e \frac{1-\varphi^{n-e+1}}{1-\varphi^{n+1}}$, $\Phi_B = \Phi + Q_e \frac{\varphi^{n-e+1}-\varphi^{n+1}}{1-\varphi^{n+1}}$, $e > m$: $\Phi_m = \Phi_A$; $T_m = \left[(T'_m)^4 + \frac{Q_e}{\sigma_0 \varepsilon_{eq} F_B} \frac{(\varphi^{n-m+1}-\varphi^{n+1})(1-\varphi^{n-e+1})}{(1-\varphi^{n+1})(1-\varphi)}\right]^{\frac{1}{4}}$, $e < m$: $\Phi_m = \Phi_B$; $T_m = \left[(T'_m)^4 + \frac{Q_e}{\sigma_0 \varepsilon_{eq} F_B} \frac{(\varphi^{n-e+1}-\varphi^{n+1})(1-\varphi^{n-m+1})}{(1-\varphi^{n+1})(1-\varphi)}\right]^{\frac{1}{4}}$
$Q_1 \neq \dots \neq Q_n \neq 0$	$\Phi_m = \Phi + \sum_{k=1}^{m-1} Q_k \frac{\varphi^{n-k+1}-\varphi^{n+1}}{1-\varphi^{n+1}} - \sum_{k=m}^{n} Q_k \frac{1-\varphi^{n-k+1}}{1-\varphi^{n+1}}$, $T_m = \left[(T'_m)^4 + \frac{1}{\sigma_0 \varepsilon_{eq} F_B}\left(\frac{1-\varphi^{n-m+1}}{1-\varphi} \sum_{k=1}^{m-1} Q_k \frac{\varphi^{n-k+1}-\varphi^{n+1}}{1-\varphi^{n+1}} + \frac{\varphi^{n-m+1}-\varphi^{n+1}}{1-\varphi} \sum_{k=m}^{n} Q_k \frac{1-\varphi^{n-k+1}}{1-\varphi^{n+1}}\right)\right]^{\frac{1}{4}}$, $\Phi_A = \Phi - \sum_{k=1}^{n} Q_k \frac{1-\varphi^{n-k+1}}{1-\varphi^{n+1}}$, $\Phi_B = \Phi + \sum_{k=1}^{n} Q_k \frac{\varphi^{n-k+1}-\varphi^{n+1}}{1-\varphi^{n+1}}$

The resultant heat flux transmitted to any shield is a function of the shield location, in the general case written down as

$$Q_k = \int_V q_{v,k}(x,\ y,\ z)\, dv.$$

a) If $Q_1 = \dots = Q_m = \dots = Q_n = 0$, we are dealing with "floating" shields. Here $\Phi_A = \dots = \Phi_m = \dots = \Phi_n = \Phi$ and (3), (6), and (7) are replaced by the same single expression

$$\Phi = \sigma_0 \varepsilon_{AB} H_{AB} (T_A^4 - T_B^4),$$

while Eq. (4) becomes

$$T'_m = \left(\frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{mB} H_{mB}} T_A^4 + \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{Am} H_{Am}} T_B^4 \right)^{\frac{1}{4}}.$$

Taking all this into account, the general Eqs. (3), (4), (6), and (7) can be rewritten as follows:

$$\Phi_m = \Phi + \sum_{k=1}^{m-1} Q_k \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{Ak} H_{Ak}} - \sum_{k=m}^{n} Q_k \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{kB} H_{kB}}, \tag{3'}$$

$$T_m = \left[(T'_m)^4 + \frac{\sum_{k=1}^{m-1} Q_k \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{Ak} H_{Ak}}}{\sigma_0 \varepsilon_{mB} H_{mB}} + \frac{\sum_{k=m}^{n} Q_k \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{kB} H_{kB}}}{\sigma_0 \varepsilon_{Am} H_{Am}} \right]^{\frac{1}{4}}, \tag{4'}$$

$$\Phi_A = \Phi - \sum_{k=1}^{n} Q_k \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{kB} H_{kB}}, \tag{6'}$$

$$\Phi_B = \Phi + \sum_{k=1}^{n} Q_k \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{Ak} H_{Ak}}. \tag{7'}$$

Here Φ and T'_m are the resultant radiative heat flux and the temperature of shield m at the same initial conditions, only now the shields are immune to the influence of extraneous sources of energy.

b) If $Q_1 = \ldots = Q_m = \ldots = Q_n = Q$, i.e., the shields receive equal heat fluxes from extraneous sources of energy, then Eqs. (3), (4), (6), and (7) become, after respective transformations,

$$\Phi_m = \Phi + Q \varepsilon_{AB} H_{AB} \sum_{i=1}^{n+1} \frac{m-i}{\varepsilon_{\mathrm{eq},i} H_i},$$

$$T_m = \left[(T'_m)^4 + \frac{Q}{\sigma_0} \left(\frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{mB} H_{mB}} \sum_{i=1}^{m-1} \frac{m-i}{\varepsilon_{\mathrm{eq},i} H_i} + \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{Am} H_{Am}} \sum_{i=m+1}^{n+1} \frac{i-m}{\varepsilon_{\mathrm{eq},i} H_i} \right) \right]^{\frac{1}{4}},$$

$$\Phi_A = \Phi - Q \varepsilon_{AB} H_{AB} \sum_{i=2}^{n+1} \frac{i-1}{\varepsilon_{\mathrm{eq},i} H_i},$$

$$\Phi_B = \Phi + Q \varepsilon_{AB} H_{AB} \sum_{i=1}^{n} \frac{n-i+1}{\varepsilon_{\mathrm{eq},i} H_i}.$$

c) If $Q_1 = \ldots = Q_{e-1} = Q_{e+1} = \ldots = Q_n = 0$ while $Q_e \neq 0$, then

$$\Phi_A = \Phi - Q_e \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{eB} H_{eB}} \quad \text{and} \quad \Phi_B = \Phi + Q_e \frac{\varepsilon_{AB} H_{AB}}{\varepsilon_{Ae} H_{Ae}},$$

while

$$\Phi_m = \Phi_A \quad \text{и} \quad T_m = \left[(T'_m)^4 + \frac{Q_e \varepsilon_{AB} H_{AB}}{\sigma_0 \varepsilon_{Am} H_{Am} \varepsilon_{eB} H_{eB}} \right]^{\frac{1}{4}},$$

for $e \geq m$, and

$$\Phi_m = \Phi_B, \quad \text{a} \quad T_m = \left[(T'_m)^4 + \frac{Q_e \varepsilon_{AB} H_{AB}}{\sigma_0 \varepsilon_{mB} H_{mB} \varepsilon_{Ae} H_{Ae}} \right]^{\frac{1}{4}},$$

for $e < m$.

Let us consider some of the most common practical cases.

1. A and B are parallel planes, very large as compared to the distance between them: $\varepsilon_A = \ldots = \varepsilon'_m = \varepsilon''_m = \ldots = \varepsilon_B = \varepsilon$ and $F_A = \ldots = F'_m = F''_m = \ldots = F_B = F$.

In this case

$$\varphi_{A1} = \varphi_{1A} = \cdots = \varphi_{nB} = \varphi_{Bn} = 1; \quad H_1 = \cdots = H_{n+1} = H_{AB} = \cdots = H_{kB} = F$$

$$\text{and} \quad \varepsilon_{eq,1} = \cdots = \varepsilon_{eq,n+1} = \varepsilon_{eq} = \frac{\varepsilon}{2-\varepsilon}.$$

Substitution into Eq. (5) yields:

$$\varepsilon_{AB} = \frac{1}{n+1}\varepsilon_{eq}, \quad \varepsilon_{Am} = \frac{1}{m}\varepsilon_{eq}, \quad \varepsilon_{mB} = \frac{1}{n-m+1}\varepsilon_{eq}, \quad \text{etc.}$$

These expressions are valid for diffusely reflecting as well as for mirror surfaces.

In Table 1 are given calculation formulas for this case.

2. Body A and shields 1 to n are located inside body B (see the picture in Table 2) in such a way that no inner surface of any closed system thus formed has concavities: $\varepsilon_A = \ldots = \varepsilon'_m = \varepsilon''_m = \ldots = \varepsilon_B = \varepsilon$; $F'_m = F''_m = F_m$ and $F_A/F_1 = F_1/F_2 = \ldots = F_n/F_B = \varphi$.

If the surfaces are diffusely reflecting, then $\varphi_{A1} = \varphi_{12} = \ldots = \varphi_{nB} = 1$, $\varphi_{1A} = \varphi_{21} = \ldots = \varphi_{Bn} = \varphi$, $H_1 = H_{AB} = H_{Am} = F_A$, $H_m = F_{m-1}$, $H_{mB} = F_m$ etc., and $\varepsilon_{eq,1} = \ldots = \varepsilon_{eq,n+1} = \varepsilon_{eq} = \varepsilon/(1 + (1-\varepsilon)\varphi)$.

If the surface are mirrors in the shape of concentric spheres or long coaxial cylinders, then their emissivities will be [6]:

$$\varepsilon_{eq,1} = \cdots = \varepsilon_{eq,n+1} = \varepsilon_{eq} = \frac{\varepsilon}{2-\varepsilon},$$

and their mutually-irradiating surface areas will be the same.

Inserting into Eq. (5) we have

$$\varepsilon_{AB} = \frac{1-\varphi}{1-\varphi^{n+1}}\varepsilon_{eq}; \quad \varepsilon_{Am} = \frac{1-\varphi}{1-\varphi^{m}}\varepsilon_{eq}; \quad \varepsilon_{mB} = \frac{1-\varphi}{1-\varphi^{n-m+1}}\varepsilon_{eq} \text{etc.}$$

In Table 2 are given calculation formulas for this case.

3. Body B and the shields for n to 1 are inside body A (see the picture in Table 3) in such a way that no inner surface of any closed system thus formed has concavities: $\varepsilon_A = \ldots = \varepsilon'_m = \varepsilon''_m = \ldots = \varepsilon_B = \varepsilon$; $F'_m = F''_m = F_m$ and $F_1/F_A = F_2/F_1 = \ldots = F_B/F_n = \varphi$.

In this case $\varphi_{A1} = \varphi_{12} = \ldots = \varphi_{nB} = \varphi$; $\varphi_{1A} = \varphi_{21} = \ldots = \varphi_{Bn} = 1$; $H_1 = F_1$; $H_m = F_m$; $H_{AB} = F_B$; $H_{Am} = F_m$; $H_{mB} = F_B$; $H_{Ak} = F_k$ etc., and the given emissivities are the same as in case 2.

In Table 3 are given calculation formulas for this case.

The equations derived here make it possible to examine the phenomenon of radiative heat transfer and to determine the conditions of most effective shielding in a closed system which comprises two surfaces of any shape, when the shields placed between these surfaces can be subject to additional heat effects from extraneous sources of energy.

NOTATION

$T_A, T_B, F_A, F_B, \varepsilon_A, \varepsilon_B$	are the temperatures (°K), the areas (m²), and the emissivities of surfaces A and B;
T_m	is the temperature of shield m;
T'_m	is the temperature of the same shield without extraneous influences;
F'_m, ε'_m	are the area and the emissivity of a shield surface exposed to A;
F''_m, ε''_m	are the area and the emissivity of a shield surface exposed to B;
$\varepsilon_{eq,i}$	is the equivalent emissivity of a closed system i;
ε_{ik}	is the equivalent emissivity of closed systems i-k;
φ_{ik}	is the mean angular coefficient of radiation from surface F_i to surface F_k;
H_{ik}	is the area of a mutually-irradiating surface in a closed system ik without any other bodies present;
Q_k	is the net heat flux to a shield, positive (kcal/h);

Φ_m is the net radiative heat flux in a closed system m (flux to B positive, flux to A negative) (kcal/h);
σ_0 is the absolute black-body radiation constant, $4.9 \cdot 10^{-8}$ kcal/m$^2 \cdot$ h $\cdot$ deg^4;
$q_{v,k}$ is the specific heat flux transmitted to unit volume k of a shield from extraneous sources of energy;
x, y, z are the space coordinates.

LITERATURE CITED

1. S.S. Kutateladze, Fundamentals of Heat Transfer Theory [in Russian], Izd. Mashgiz, Moscow (1957).
2. M.P. Malkov, I.B. Danilov, A.G. Zel'dovich, and A.B. Fradkov, Symp. on Physics and Engineering Fundamentals of Cryogenics [in Russian], Gosénergoizdat (1963).
3. P.L. Privalov, Pribory i Tekh. Éksperim., No.4, 150 (1967).
4. A.W. Lawson and R. Fano, Rev. Sci. Instrum., 18, 727 (1947).
5. J.B. Garrison and A.W. Lawson, Rev. Sci. Instrum., 19, 574 (1948).
6. P.B. Scott, Low-Temperatures Engineering [Russian translation], IL (1962).